Energy Efficient Control of Robots with Variable Stiffness Actuators

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Abstract: Variable stiffness actuators realize a particular class of actuators characterized by the property that the apparent output stiffness can be changed independently of the output position. This is feasible due to the presence of internal springs and internal actuated degrees of freedom. In this work, we establish a port-based model of variable stiffness actuators and we derive an energy efficient control strategy. In particular, when the variable stiffness actuator acts on a mechanical system, the internal degrees of freedom are used to achieve the desired nominal behavior and the internal springs are used as a potential energy buffer. The release of energy from the springs, as well as the apparent output stiffness, are regulated by control of the internal degrees of freedom. Simulation results on a robotic joint illustrate the effectiveness of the control strategy during the tracking of a periodic motion in presence of disturbances.

Keywords: Variable stiffness actuators, Energy efficient control, Mathematical models, Nonlinear analysis, Robotics

1. INTRODUCTION

The research interests and efforts in variable stiffness actuators are increasing in recent years due to their broad range of possible applications. The main characteristic property of a variable stiffness actuator is that the output stiffness can be varied independently from the output position, thanks to the presence of internal actuated degrees of freedom and internal springs. This means that, if the joints of a robot are actuated by means of this class of actuators, it is possible for the robot to perform different tasks while appearing more or less compliant.

Recently, various designs of variable stiffness actuators have been introduced, for example AMASC (Hurst, 2004), VSA (Tonietti, 2005), VS-Joint (Wolf, 2008) and MACCEPA (Vanderborght, 2009). All these actuators use a number of internal springs with a fixed elastic constant, and the output stiffness is varied by changing the configuration of some internal degrees of freedom. In the context of safe interaction, the mechanical compliance is controlled only if an unexpected collision occurs and it is used to reduce the impact force (De Luca, 2009).

Besides using the internal springs of a variable stiffness actuator solely to introduce a mechanical compliance to the joints for safety reasons (Bicchi, 2004), they can also be exploited to achieve more energy efficient actuation by storing negative work (Stramigioli, 2008) or to change the natural frequencies of the system to match the periodicity of the motion (Uemura, 2009). In recent work, we presented a novel energy efficient variable stiffness actuator, characterized by the property that the output position and output stiffness are decoupled on a mechanical level (Visser, 2010a). This property allows the internal springs to be used as buffers to temporarily store potential energy. For example, when a disturbance occurs, the springs can store the disturbance energy, which then can be reused to bring the robot back to the desired trajectory. In particular, this approach can have big advantages in trajectory tracking (Li and Horowitz, 1999; Duindam, 2004). In such applications, the desired joint trajectories do not depend on time, and hence the potential energy stored in the springs can be used efficiently. This approach can be beneficial to walking robots, where energy efficiency is of paramount concern and trajectory tracking controllers can improve the robustness (Duindam and Stramigioli, 2009).

Building on our previous work, in this paper we establish a formal, port-based mathematical model for the analysis and the control of variable stiffness actuators. The port-based framework not only provides valuable insights on energy flows between the internal actuators, the springs and the robot, but it is also a solid foundation for research on innovative control methods. With the aim of achieving energy efficiency, we derive a control architecture that uses the potential energy stored in the springs to actuate the robot, instead of supplying this energy by controlling the internal degrees of freedom. We demonstrate the effectiveness of the proposed controller for a one degree of freedom system, under influence of a disturbance. In particular, if a disturbance occurs, the corresponding energy is stored in the internal springs and used for actuation. The insights gained will form a basis for future work, in which we aim to develop energy efficient coordinated control methods for a robot with multiple degrees of freedom.

* This work has been funded by the European Commission’s Seventh Framework Programme as part of the project VIACTORS under grant no. 231554.
2. PORT-BASED MODELING OF VARIABLE STIFFNESS ACTUATORS

In this Section, we intend to briefly recall the port-based generalized model of variable stiffness actuators, introduced in our previous work (Visser, 2010a,b), and to provide a more solid mathematical foundation, which is the basis for both the present paper and future research. The port-based framework gives valuable insights in the power flows between the controller, the variable stiffness actuator and the actuated system. Therefore, it realizes an appropriate tool for the analysis, modeling and control of systems in which energy efficiency is the main concern.

Without loss of generality, we assume that a variable stiffness actuator has the following properties:

- \( n \geq 2 \) internal degrees of freedom, denoted by \( q \in Q \), can be actuated;
- \( m \geq 1 \) springs, either linear or nonlinear, are internally present;
- the apparent output stiffness \( K \) of the actuator depends on both the configuration of the internal degrees of freedom and of the internal springs.

Since the aim of the model is to analyze the functional principle of a variable stiffness actuator, rather than evaluating the mechanical design, internal friction and inertias are neglected. The generic model of a variable stiffness actuator is depicted in Figure 1, using a bond graph representation. Each bond represents a power flow, defined positive in the direction of the half arrow and characterized by power conjugate variables \( \tau \) and \( F \), i.e. the generalized forces \( \tau \) and \( F \) and the output position \( x \) are subject to the dynamics \( \dot{q} = u_q, \dot{x} = u_x \) given by

\[
\dot{q} = \sum_{i=1}^{n} v_{q,i} u_{q,i}, \quad \dot{x} = v_x u_x
\]

Equation (1) can be written as

\[
v_{q,i} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T
\]

\[
v_x = 1
\]

define constant input vector fields on \( TQ \times TX \), i.e. the canonical basis for the tangent space. Since these dynamics are trivial, we allow abuse of notation and consider \( \dot{q} \) and \( \dot{x} \) as inputs to the system. These relations are summarized in the commutative diagram in Figure 2.

The dynamics of the variable stiffness actuator are

\[
\delta \dot{s} = \Gamma_{q,x}^* (\dot{q}, \dot{x})
\]

\[
(\tau, F) = \Gamma_{q,x}^* (dH)
\]

where \( dH \) denotes the differential of the energy function \( H \). We can see that flows are elements of the tangent spaces and effort elements of the cotangent spaces. The Dirac structure is in the tangent maps \( \Gamma_{q} : TQ \to TS \), \( \Gamma_{x} : TX \to TS \) and the corresponding cotangent maps \( \Gamma_{q}^* \Gamma_{x}^* \). Via these maps, the velocities on \( T_sS \) at \( s \in S \) are
In this Section, an energy efficient control law for variable stiffness actuators is derived. This means that, in order to achieve the energy efficiency of the controller, we intend to use the energy stored in the internal springs, if present, for the actuation of the robotic joint. First, we formulate the problem statements and, then, derive a solution that accomplishes the energy efficiency requirements. Moreover, since the fundamental demand of variable stiffness actuators is the capability of changing the output stiffness, we extend the control law with stiffness regulation.

3.1 Problem Statement

As depicted in Figure 1, the variable stiffness actuator is connected to a robotic joint with one degree of freedom. This connection is explicitly shown in Figure 3 by a 1-junction representing the power continuous connection. Assume that full state measurement is available. How can the control inputs \( \dot{q} \) to the variable stiffness actuator be designed such that the desired joint input \( u_d \) is achieved in an energy efficient way?

3.2 Output Force Control

Before proceeding with the formulation of the control law, it is necessary to highlight that, if a desired input force (torque) \( u_d \) is required at the robotic joint, also the desired output force \( F_d \) of the actuator is known due to the power continuous connection. Moreover, from (4), the output force \( F \) generated by the variable stiffness actuator, can be determined once the current values for \( q \) and \( x \) are given. This means that it is possible to find a control law \( g_F(\cdot) \) such that the evolution in time of the force \( F \) is given by

\[
\dot{F} = g_F(F_d, F)
\]

as a function of the actual and the desired output force. We can then formulate a nominal control law as follows.

**Lemma 3.3.** (Nominal control). Consider a variable stiffness actuator described by an energy function \( H \) and the dynamics (4), connected to the joint of a robot through a power continuous connection. Let the vector valued function \( V(q, x) \) be:

\[
V(q, x) := (L_{v_{q,1}} F, \ldots, L_{v_{q,n}} F)
\]

where \( L_{v_{q,i}} F \) denotes the Lie-derivative of \( F \) along the vector field \( v_{q,i} \). Define a subset \( \mathcal{M} \) of \( \mathcal{Q} \times \mathcal{X} \) as:

\[
\mathcal{M} = \{(q, x) \in \mathcal{Q} \times \mathcal{X} \mid V(q, x) \neq 0\}
\]
By using the control law \( \dot{F} = g_F(F_d, F) \), the nominal control input

\[
\dot{q}_n = V^+ \left( \dot{F} - (L_{v_x} F) \dot{x} \right)
\]

(8)

where \( + \) denotes the Moore-Penrose pseudo inverse, solves Problem 3.1 for \((q, x)\) in \( M \).

**Proof:** Since we defined the exact one-form \( \cdot \mathcal{H} \) on \( T_q^* S \), we have that \((\tau, F)\) is an exact one-form on \( T_q^* Q \times T_q^* X \), defined by the map \( \Gamma_{(q,x)} \) (Nijmeijer and van der Schaft, 1990). Moreover, since the input vector fields, as defined in (2) and (3) are constant, the rate of change of the one-form \((\tau, F)\) is given by

\[
\frac{d}{dt}(\tau, F) = (\dot{q}_1 \cdots \dot{q}_n) \left( L_{v_{q,1}}(\tau, F) \right) \cdots \left( L_{v_{q,n}}(\tau, F) \right)
\]

(9)

where we allowed some abuse of notation, as in (1). Since \( \dim X = 1 \), from (9) it follows that:

\[
\dot{F} = V \dot{q} + (L_{v_x} F) \dot{x}
\]

(10)

with \( V \) given in (7). Finally, from (10), the nominal controller \( \dot{q}_n \) in (8) follows.

Note that the restriction of the solution to \( M \) follows from the Moore-Penrose pseudo inverse for the full row-rank matrix \( V \) (Ben-Israel and Greville, 2003):

\[
V^+ = V^T (VV^T)^{-1}
\]

i.e., the pseudo inverse is only defined for \((q, x)\) in \( M \). \( \Box \)

### 3.3 Energy Efficient Control

The solution derived in Lemma 3.3 is not energy efficient, since the energy stored in the springs is not taken into account. If a disturbance is present on the output port, we intend to direct the corresponding energy to the springs so to use them as energy buffer. Since our goal is to obtain solutions that efficiently use energy stored in the springs, if there is energy stored in the springs, there is, in general, no need to supply more energy via the control port.

For a particular class of variable stiffness actuators, the solution (8) can be modified such that any potential change due to the control input. Hence, the solutions to Problem 3.1 given by (8) should be in ker \( \Gamma_{q,s} \), or close to it when energy is stored in the springs. This can be achieved by defining on \( T_q Q \) an appropriate metric that weights the solutions given by the Moore-Penrose pseudo inverse (Ben-Israel and Greville, 2003). This argument is formalized in the following Lemma.

**Lemma 3.4.** (Energy efficient control). Consider a variable stiffness actuator described by an energy function \( H \) and the dynamics (4) and connected to the joint of a robot by a power continuous connection. Assume that the variable stiffness actuator satisfies the property

\[
\dim \ker \Gamma_{q,s} = k, \quad 1 \leq k < n, \quad \forall(q, x) \in Q \times X
\]

Take on \( T_q Q \) two sets of local coordinates, orthogonal in the Euclidean sense, denoted by \( a^1 = (a^1_1, \ldots, a^1_k) \) and \( a^2 = (a^2_1, \ldots, a^2_{n-k}) \), satisfying

\[
\ker \Gamma_{q,s} = \text{span} \{a^1\}
\]

\[
D = \text{span} \{a^2\}
\]

(11)

On \( T_q Q \), define a metric \( g \), such that in the local coordinates \((a^1, a^2)\) its components \([g]\) are given by

\[
[g] = \begin{bmatrix} I_k & 0 \\ 0 & \alpha I_{n-k} \end{bmatrix}
\]

(12)

with \( I_k \) and \( I_{n-k} \) the identity matrix of dimension \( k \) and \( n - k \) respectively, and \( \alpha : S \rightarrow \mathbb{R}^+ \) a positive definite function realizing a measure for the amount of energy stored in the springs.

Then, for \((q, x)\) in the subset \( M \), the control input

\[
\dot{q}_c = V^2 \left( \dot{F} - (L_{v_x} F) \dot{x} \right)
\]

(13)

where \( \dot{z} \) denotes the Moore-Penrose pseudo inverse with respect to the metric \( g \) defined in (12), solves Problem 3.1 in an energy efficient way by exploiting the energy stored in the springs. \( \Box \)

**Proof:** Using the partitioning (11), the solution (8) can be expressed into components that are either in ker \( \Gamma_{q,s} \), or outside, i.e. in \( D \). Figure 4 depicts a two dimensional example, in which \( \left( \frac{a}{\dot{a}}, \frac{\dot{a}}{\dot{a}} \right) \) denotes the canonical basis for \( T_q Q \), the one-dimensional spaces ker \( \Gamma_{q,s} \), and \( D \) are spanned by the vectors \( a^1 \) and \( a^2 \), respectively. By choosing the metric \( g \) as proposed in (12), the components of the solution \( \dot{q}_c \) in \( D \) are weighted by \( \alpha \). In particular, since \( \alpha \) is chosen such that it is proportional to the energy stored in the springs, the component in \( D \) is smaller when there is more energy available in the springs. Hence, from (5) it follows that, when there is more energy available in the springs, less energy is supplied via the control port. \( \Box \)

### 3.4 Stiffness Control

The control law derived in Lemma 3.4 achieves control of the joint position, but does not control the apparent stiffness of the joint. However, in many applications it is desired that the apparent joint stiffness attains some specific value. Therefore, we extend the control law with stiffness control. In particular, because of the redundancy in the internal degrees of freedom, we can define a control input, to add to any solution of Problem 3.1, so that the stiffness is controlled independently from the generated.
output force. This requires that the additional stiffness control \( \dot{q}_V \in \ker V \), so to not affect the control \( \dot{q}_e \) in (13).

Under the assumption of full state measurement, the apparent output stiffness \( K \) of the actuator may be estimated. Given a desired output stiffness \( K_d \), we can design a control law \( g_K(\cdot) \) such that the desired rate of change of the stiffness is given by

\[
\dot{K} = g_K(K_d, K)
\]

If we model the stiffness \( K \) as a function on the configuration manifold, i.e., \( K : Q \times X \to \mathbb{R} \), we can formulate a control law for the stiffness as follows.

**Lemma 3.5.** (Stiffness control). Define on \( T_q Q \) two sets of coordinates, denoted by \( b^1 \) and \( b^2 \), satisfying:

\[
\ker V = \text{span}\{b^1\}
\]

\[
T_q Q = \text{span}\{b^1, b^2\}
\]

Determine a solution \( \dot{q}_k \) that achieves the desired rate of change of the stiffness, i.e. a solution satisfying

\[
(L_{v_{q,i}} K \cdots L_{v_{q,n}} K) \ \dot{q}_k = g_K(K_d, K)
\]

where \( L_{v_{q,i}} K \) denotes the Lie-derivative of \( K \) along \( v_{q,i} \). Denote by \( \dot{q}_V \) the projection of \( \dot{q}_k \) onto \( b^1 \). Then, the solution to Problem 3.2 is given by the control input

\[
\dot{q}_k = \dot{q}_e + \dot{q}_V
\]

**Proof:** The control input \( \dot{q}_k \) is chosen such that the stiffness changes as desired, and by taking the projection onto \( \ker V \), the stiffness is changed while Problem 3.1 is still solved.

**Remark 3.6.** By taking the projection onto \( \ker V \), it is ensured that at all times Problem 3.1 is solved. However, it follows that, in general, \( \dot{q}_V \neq \dot{q}_k \), and thus that the stiffness does not change exactly as desired, but as close to desired as possible.

**Remark 3.7.** Since \( \dot{q}_V \) was not obtained with respect to the metric \( g \) as defined in Lemma 3.4, the solution (14) is not necessarily energy efficient. In particular, the choice of \( g_K(\cdot) \) determines the component outside \( \ker \Gamma_{q^*} \).

4. SIMULATION RESULTS

In this Section, we show the effectiveness of the control law derived in Section 3. This will be done by using a linear variable stiffness actuator design, presented in earlier work (Visser, 2010a), which satisfies the condition stated in Lemma 3.4, to actuate a joint. The experiment is as follows. The actuator moves a linear joint on a periodic motion following a sinusoidal trajectory with an amplitude of 10 cm at a frequency of 0.1 Hz. The force \( u_d \) needed to make the joint follow the trajectory is calculated using a PD-control law using the current and desired position and velocity. In the time interval \( 5 \leq t \leq 6 \) s, the joint is subjected to a 2 N constant disturbance force. The same experiment is performed with each of the three presented control laws (8), (13), (14), and their performance is compared. In particular, it is investigated how each of the controllers handles the disturbance energy. The results are presented in Figures 5-7.

Figure 5 shows the phase space trajectory of the joint. It can be seen that the response to the disturbance is similar for each of the controllers. Note that each controller required approximately the same amount of time to return to the desired trajectory. In Figure 6, the energy supplied via the control port of the variable stiffness actuator is plotted. In particular, the absolute power flow through the control port is integrated, to make explicit that negative work is lost. From the numerical values, it can be seen that the energy efficient control law indeed achieves a sig-
significant reduction in energy consumption (approximately 9.6% with respect to the nominal control law). When stiffness regulation is added, the reduction in energy consumption is less (approximately 2.6%), as was expected. From Figure 7, the added benefit of stiffness regulation can be seen. Initially, the output stiffness is kept to a desired value of 200 N/m. When the disturbance occurs, the output stiffness increases, because the internal springs store the disturbance energy. The increase in stiffness might be an undesirable side effect in some applications, e.g. in human-robot interaction, and thus the potential benefit of adding stiffness regulation is illustrated. However, at the same time, it is illustrated that energy efficient control and regulating stiffness are contradicting goals.

5. CONCLUSIONS AND FUTURE WORK

In this work, we presented an energy efficient control method for variable stiffness actuators. In addition, a stiffness regulation control was implemented, with the aim of maintaining a desired output stiffness. Simulation results illustrate the effectiveness of the proposed method. In particular, it was shown that the energy efficient control law indeed achieves a significant reduction in the energy supplied via the control port of the variable stiffness actuator by reusing energy stored in the springs. Adding stiffness regulation reduces the energy efficiency, but the controller still performs better than the nominal controller. It was found that the strict time dependency of the reference trajectory in the simulation can sometimes cause the controllers to perform poorly. This is due to the inherent oscillatory behaviour of the springs. We believe that, in limit cycle trajectory tracking applications, the advantages of the proposed control strategy will become more apparent. Since in such trajectory tracking application time is no longer restrictive, the controller may perform better by taking a state dependent response to the disturbance.

Future work will focus on controlling multiple degree of freedom systems under influence of significant disturbances, where the energy storing capabilities of the springs are more useful. The aim is to control the joints in a coordinated way, and come to an energy efficient disturbance correction. In particular, in the control of walking robots, the energy losses, associated with the impacts of the feet, can be reduced using our proposed control method.

REFERENCES